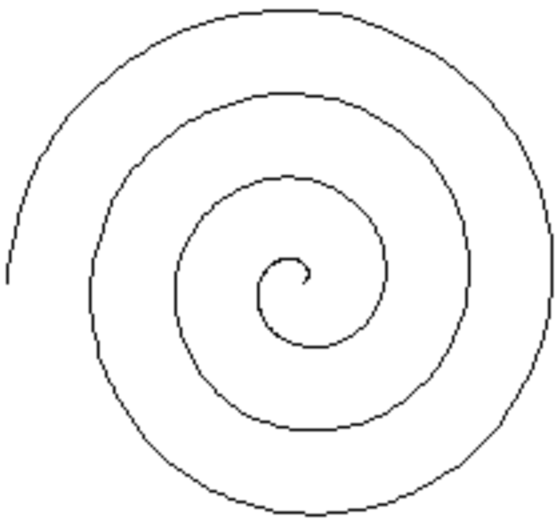
Archimedean Spirals *

An Archimedean Spiral is a curve defined by a polar equation of the form $r = \theta^a$. Special names are being given for certain values of a. For example if $a = 1$, so $r = \theta$, then it is called Archimedes' Spiral.

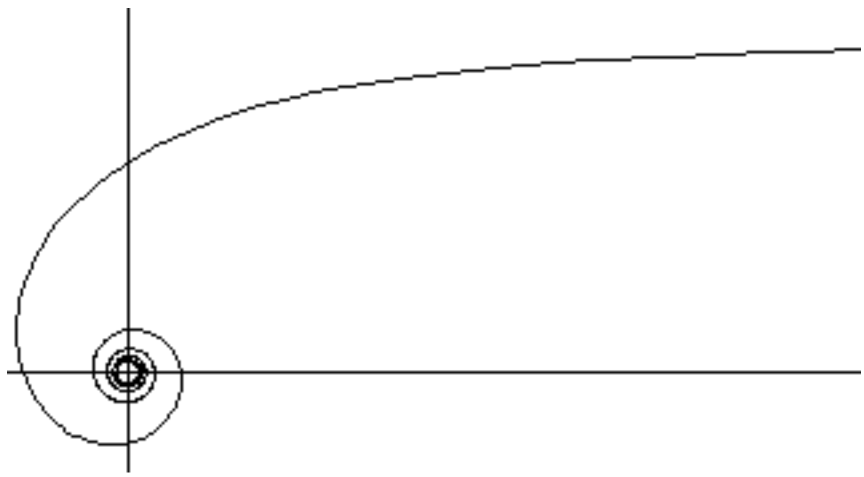


Archimede's Spiral

Formulas in 3DXM:

$r(t) := t^{aa}$, $\theta(t) := t$,
Default Morph:
$-1 \leq aa \leq 1.25$.

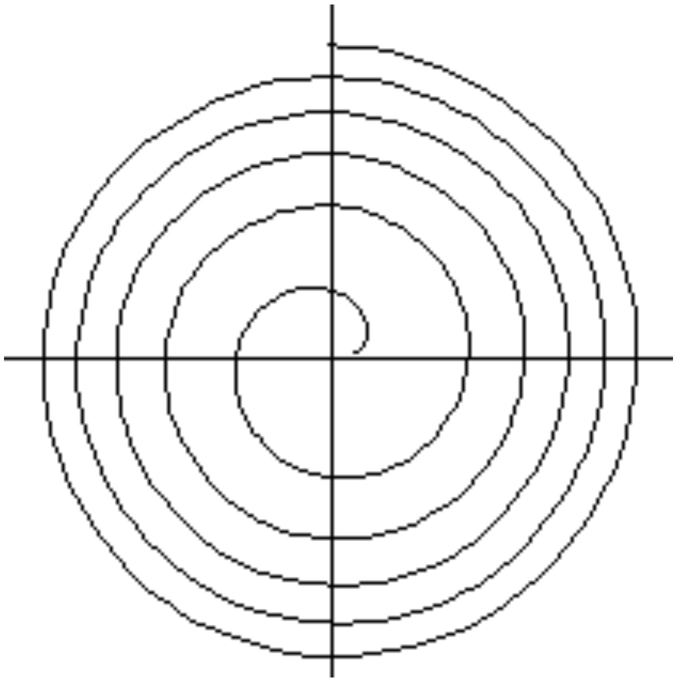
For $a = -1$, so $r = 1/\theta$, we get the reciprocal (or hyperbolic) spiral:



Reciprocal Spiral

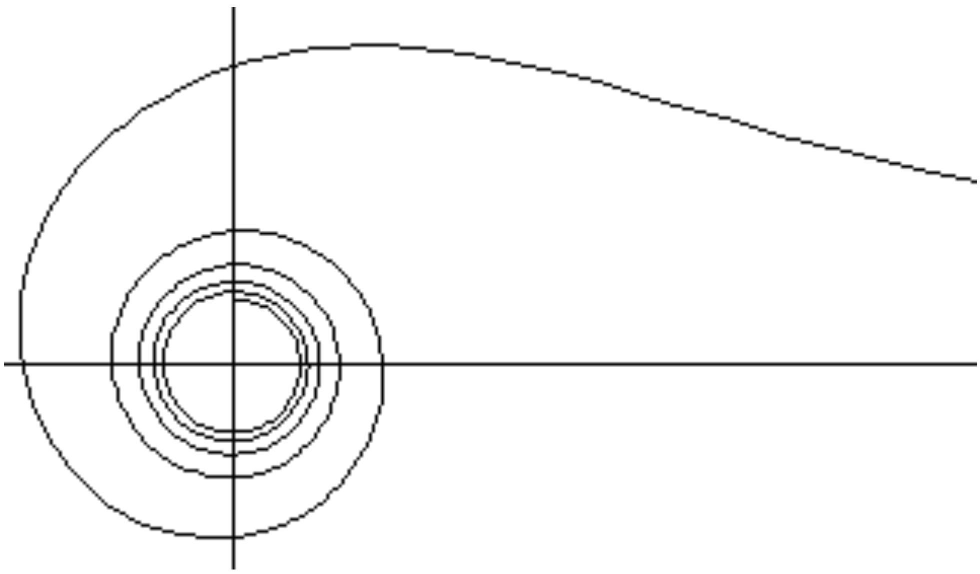
* This file is from the 3D-XplorMath project. Please see: http://3D-XplorMath.org/

The case $a = 1/2$, so $r = \sqrt{\theta}$, is called the Fermat (or hyperbolic) spiral.



Fermat's Spiral

While $a = -1/2$, or $r = 1/\sqrt{\theta}$, it is called the Lituus:



Lituus

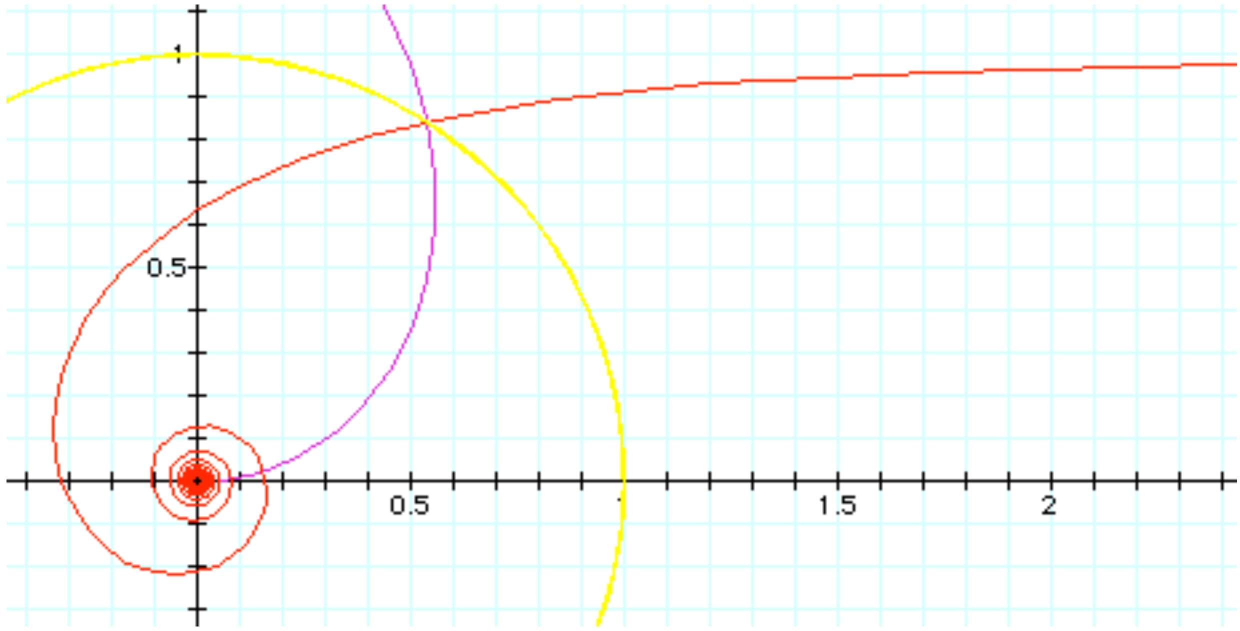
In 3D-XplorMath, you can change the parameter a by going to the menu Settings $\rightarrow$ Set Parameters, and change the value of aa. You can see an animation of Archimedean spirals where the exponent $a = aa$ varies gradually, between -1 and 1.25. See the *Animate Menu*, entry *Morph*.

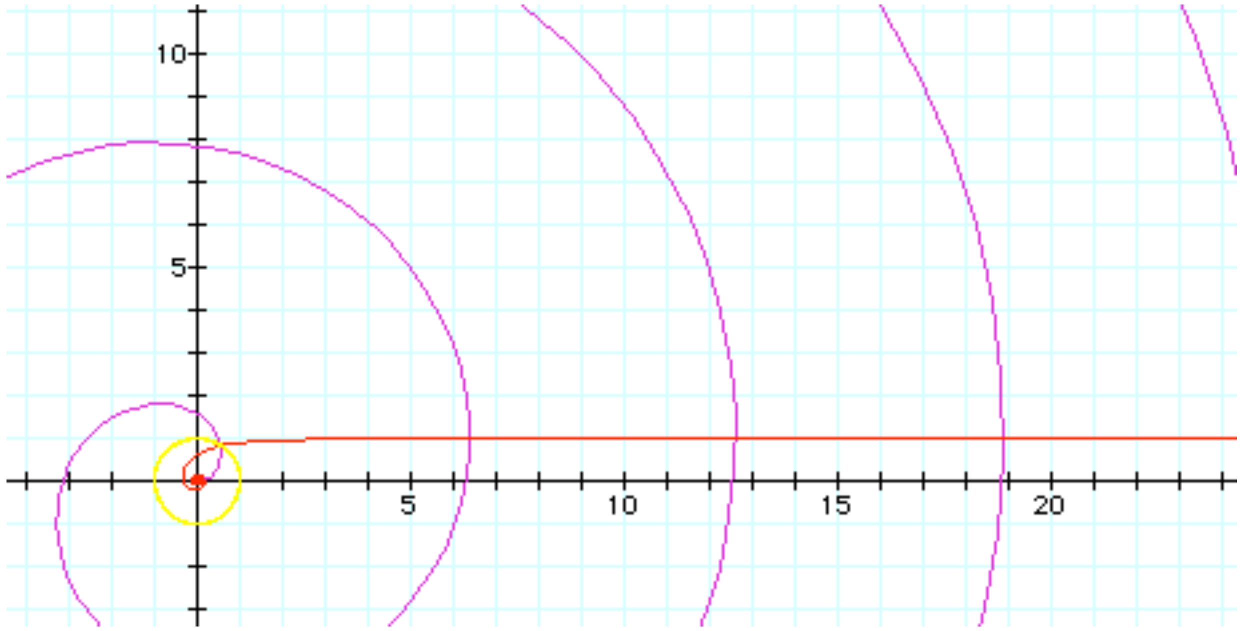
The reason that the parabolic spiral and the hyperbolic spiral are so named is that their equations in polar coordinates, $r\theta = 1$ and $r^2 = \theta$, respectively resembles the equations for a hyperbola ($xy = 1$) and parabola ($x^2 = y$) in rectangular coordinates.

The hyperbolic spiral is also called reciprocal spiral because it is the inverse curve of Archimedes' spiral, with inversion center at the origin.

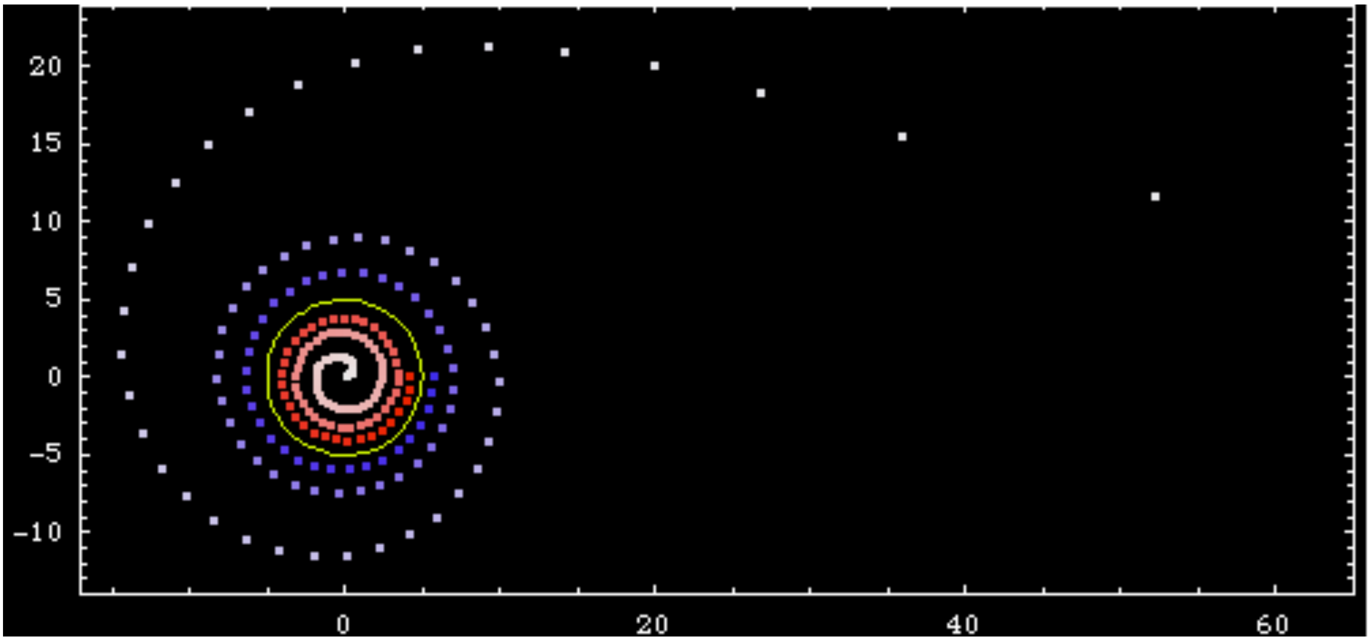
The inversion curve of any Archimedean spirals with respect to a circle as center is another Archimedean spiral, scaled by the square of the radius of the circle. This is easily seen as follows. If a point P in the plane has polar coordinates (r, θ), then under inversion in the circle of radius b centered at the origin, it gets mapped to the point P' with polar coordinates $(b^2/r, \theta)$, so that points having polar coordinates (t^a, θ) are mapped to points having polar coordinates $(b^2 t^{-a}, \theta)$.
From the above, we can see that the Archimedes' spiral inverts to the reciprocal spiral, and Fermat's spiral inverts to the Lituus.

The following two images illustrates Archimedes's spiral and Reciprocal spiral as mutual inverses. The red curve is the reciprocal spiral, the purple is the Archimedes' spiral. The yellow is the inversion circle.





The following image illustrates a Lituus and Fermat's spiral as mutual inverses. The red curve is the Fermat's spiral. The blue curve is its inversion, which is a lituus scaled by 5^2. The yellow circle is the inversion circle with radius 5. Note that points inside the circle gets mapped to outside of the circle. The closer the point is to the origin, the farther is its corresponding point outside the circle.



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